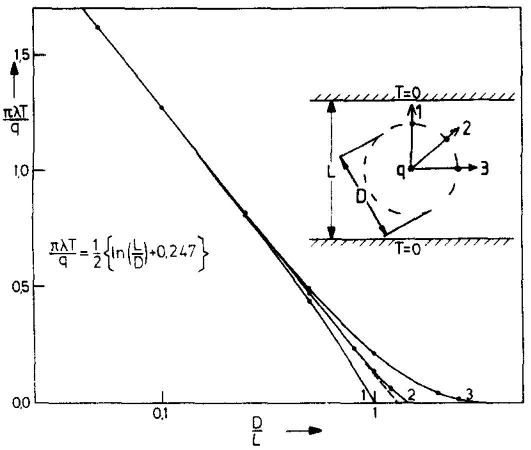

FIG. 2. Steady-state temperature for three positions at a distance $D/2$ from a continuous line source with strength $1/(\rho C_p)$, the source being positioned in the middle between two plates at zero relative temperature.

provided D_i/L is sufficiently small, i.e. <0.1. Consequently, the heat transfer is direction independent in that case as well. From the slope of the straight line in Fig. 2 it is concluded that

$$Nu = hD_i/\lambda = q/\{\pi\lambda T_1\} = 2/\ln\{1.28L/D\}.$$

This result fits much better in the expected order of decreasing Nusselt numbers for a cylinder, a quadrangular tube and a slit as enclosures.

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Effect of boundary conditions at the lateral walls on the thermal entry lengths of horizontal CVD reactors

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1. INTRODUCTION

THE PREPARATION of semi-conductors, insulators and metals from vapor deposition has lately gained increasing importance. In the production of these films for electronic and optical devices, open flow systems have become of particular interest [1]. The interest in flow profiles and possible flow instabilities stems from the need to find conditions which will improve the uniformity of the deposit. Many configurations for CVD reactors have been proposed and are in use nowadays. For instance, the horizontal CVD reactor with its simple configuration constitutes one of the reactors most amenable for the analysis of flow phenomena. Although these reactors are currently used primarily for research and special applications, they still play an important role [2]. Upon entering the heated susceptor region of the reactor, the fluid starts to heat up and will develop a new linear profile (in the absence of natural convection) within its thermal entry length, x_D. It can be shown that thermal instabilities (that is, secondary flows) will be present in this development region for all non-zero Rayleigh numbers. Indeed, two-dimensional numerical results for the GaAs hot-wall reactor showed that rather small temperature differences between successive isothermal zones can cause back-flow against the imposed forced flow [3]. These secondary flows will be confined to the entrance or will exist throughout the reactor [4]. Numerical studies by Cheng *et al.* [5], Ou *et al.* [6] and Cheng and Ou [7] showed that for the case of large Prandtl number (Pr) fluids, the inclusion of secondary flow always lead to thermal entry lengths which are shorter than lengths which are determined in the Graetz fashion (neglecting secondary flows). In this work we assume that these numerical results will hold qualitatively for gaseous systems.

The importance of knowing this distance, lies among other reasons, on the temperature distribution that will be responsible for the distribution of homogeneous gas-phase reactions, besides governing transport and physical fluid properties. From a mathematical point of view, it is advantageous to know where the linear temperature profile is established since it is the basic state that will be perturbed to find critical values of the Rayleigh number in the developed region.

Several authors treated the limiting case of two infinite horizontal plates. Hsu [8] solved the case of a step increase in heat flux in both the top and bottom plates at $x = 0$. He considered axial conduction and showed that it is important for low Peclet numbers ($Pe < 45$). Hatton and Turton [9] studied the thermal development when the top plate is at ambient temperature and the bottom plate is heated. Cheng and Wu [10] studied the influence of axial conduction on thermal instabilities in the entrance region. Hwang and Cheng [11] considered a similar problem and found that for $Pr \geqslant 0.7$ the flow is thermally more stable in the thermal

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entrance region than in the fully developed region. A three-dimensional numerical study was performed by Mahaney *et al.* [12] for mixed convection in an adiabatic duct with uniform bottom heating.

Kamotani and Ostrach [13] carried out an experimental investigation of the thermal entrance region of a horizontal parallel plate channel with adiabatic lateral walls. The fluid was air, and they found that the thermal entrance length did not change appreciably with Rayleigh number. Giling [14] used interference holography to study flow patterns in horizontal reactors. Thermal entry lengths of the heavy gases were typically eight times longer than in the case of H_2 and He. Chiu and Rosenberger [15] used laser Doppler anemometry in the study of entrance effects and fully developed mixed convection flow between horizontal plates. For the sake of simplicity and following the guidelines suggested by Rosenberger [1], we restrict our study to subcritical values of Ra (that is, $Ra < 1707$, the value associated with an infinite longitudinal aspect ratio). It is also assumed that chemically reactive species are diluted to the extent that any heat of reaction, either in the gas phase or on the heated surfaces, can be neglected.

2. THE MODEL

Consider a rectangular duct with width/height $= \eta$ with temperatures T_0 and T_s at the top and bottom surfaces. It is assumed that the isothermal fluid is hydrodynamically fully developed upon entering the reacting region. Fully developed laminar flow in a duct is given by

$$U_L(y,z) = \frac{48}{\pi^3} U_m \sum_{i=1,3,5}^{\infty} (-1)^{(i-1)/2} \times \frac{\left[1 - \dfrac{\cosh(i\pi y - i\pi\eta/2)}{\cosh(i\pi\eta/2)}\right]}{1 - \sum_i \dfrac{192}{\eta\pi^5}(\tanh(\pi i\eta/2)/i^5)} \frac{\sin(\pi i z)}{i^3} \quad (1)$$

where U_m is the average velocity in the duct (p. 123 of ref. [16]). The series in equation (1) is well approximated by the first term since it converges rapidly. Since the Galerkin method is employed in finding the primary eigenvalue associated with the slowest decay, only the denominator is of significance in the truncation. Moreover, for typical CVD configurations $\eta \geq 1$ and since the series in the denominator converges more rapidly for larger η, a one-term truncation is in order. Neglecting axial conduction, the energy balance is given by

$$\rho_0 C_p U_L \frac{\partial T}{\partial x} = \frac{k}{H}\left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad (2)$$

where ρ_0 is the reference density, C_p the specific heat and k the thermal conductivity of the fluid. The independent variables were rendered non-dimensional after dividing by the height of the reactor, H. Introducing the dimensionless temperature $\theta = (T-T_0)/(T_s-T_0)$ and the dimensionless group $Pe = \rho_0 C_p U_m H/k$, equation (2) becomes

$$\frac{48Pe}{\pi^3} \frac{\left(1 - \dfrac{\cosh(\pi y - \pi\eta/2)}{\cosh(\pi\eta/2)}\right)}{\left(1 - \dfrac{192}{\eta\pi^5}\tanh(\pi\eta/2)\right)} \sin\pi z \frac{\partial\theta}{\partial x} = \frac{\partial^2\theta}{\partial z^2} + \frac{\partial^2\theta}{\partial y^2}. \quad (3)$$

The boundary conditions are

$$\theta(0,y,z) = \frac{\partial\theta}{\partial x}(\infty,y,z) = \theta(x,y,0) - 1 = \theta(x,y,1) = 0. \quad (4)$$

The adiabatic lateral walls are

$$\frac{\partial\theta}{\partial y}(x,0,z) = \frac{\partial\theta}{\partial y}(x,\eta,z) = 0. \quad (5)$$

The conducting lateral walls are

$$\theta(x,0,z) = \theta(x,\eta,z) = 1-z. \quad (6)$$

The fully developed temperature field is given by

$$\theta_{fd} = 1 - z$$

and introducing

$$\theta_2 = \theta - \theta_{fd}$$

equations (3)–(6) become

$$Pe\frac{48}{\pi^3} \frac{\left[1 - \dfrac{\cosh(\pi y - \pi\eta/2)}{\cosh(\pi\eta/2)}\right]}{\left[1 - \dfrac{192}{\eta\pi^5}\tanh(\pi\eta/2)\right]} \sin\pi z \frac{\partial\theta_2}{\partial x} = \frac{\partial^2\theta_2}{\partial z^2} + \frac{\partial^2\theta_2}{\partial y^2} \quad (7)$$

$$\theta_2(0,y,z) - (z-1) = \frac{\partial\theta_2}{\partial x}(\infty,y,z) = \theta_2(x,y,0) = \theta_2(x,y,1) = 0. \quad (8)$$

The adiabatic case is

$$\frac{\partial\theta_2}{\partial x}(x,0,z) = \frac{\partial\theta_2}{\partial x}(x,\eta,z) = 0. \quad (9)$$

The conducting case is

$$\theta_2(x,0,z) = \theta_2(x,\eta,z) = 0. \quad (10)$$

3. THERMAL ENTRY LENGTHS

3.1. *Adiabatic lateral walls*

Since no lateral heat flux exists, the solution of the Sturm–Liouville problem, eqns (7)–(9), is sought in the form

$$\theta_2 = \sum_{n=1}^{\infty} C_{En} Y_{En}(z) \exp(-\beta_{En}^2 x) + \sum_{n=1}^{\infty} C_{On} Y_{On}(z) \exp(-\beta_{On}^2 x) \quad (11)$$

to satisfy equation (8). Subscripts E and O denote even and odd solutions. Although Y_{E1} and Y_{O1} can be found from numerical integration, they are quite adequately described by

$$Y_{E1}(z) = \sin(\pi z), \quad Y_{O1}(z) = \sin(2\pi z).$$

It is only necessary to find the primary eigenvalue, $\min(\beta_{E1}^2, \beta_{O1}^2)$, since it marks the slowest axial decay of θ_2. To show the validity of our approximation, we compare β_{E1} and β_{O1} with the values in ref. [9] for the problem

$$-\frac{d^2Y}{dy^2} + \beta^2(1-y^2)Y = 0, \quad Y(-1) = Y(1) = 0.$$

Applying the Galerkin method for our approximation gives

$$\beta_{E1} = 1.68474 (Y_{O1}(z) = \sin\pi z);$$

$$\beta_{O1} = 3.70929 (Y_{E1}(z) = \cos\pi z/2)$$

compared with the results of ref. [9]

$$\beta_{E1} = 1.68159; \quad \beta_{O1} = 3.67229.$$

Returning to equations (7)–(9), the primary eigenvalue is given by

$$\beta_{E1}^2 = \frac{\pi^2\eta}{2 \times 48/\pi^3 \times 4/3\pi \times I_1 \times Pe} \quad (12)$$

where

$$I_1 = \int_0^\eta \left[1 - \frac{\cosh(\pi y - \pi\eta/2)}{\cosh(\pi\eta/2)}\right] dy \Bigg/ \left(1 - \frac{192}{\eta\pi^5}\tanh(\pi\eta/2)\right).$$

β_{E1} is not very sensitive towards η, for $\eta \geq 1$. The thermal entrance length can now be arbitrarily defined as the distance where $|\theta_2|$ has decayed to a certain fraction of its value at $x = 0$. Following the suggestion of van de Ven *et al.* [17], this fraction can be taken as 1%, in which case the thermal entry length for $\eta = 1$ and $\eta \to \infty$ is respectively

$$x_D = 0.6Pe \text{ and } 0.612Pe.$$

Kamotani and Ostrach [13] found from experimental measurements that $x_D \approx 0.4Pe$ which corresponds with a decay of θ_2 to 10% of its original value.

3.2. *Conducting lateral walls*

In this case a lateral heat flux occurs and the solution of equations (7), (8) and (10) is sought in the following form:

$$\theta_2 = \sum_{n=1}^{\infty} C_{En} Y_{En}(x,y) \exp(-\beta_n^2 x) + \sum_{n=1}^{\infty} C_{On} Y_{On}(x,y) \exp(-\beta_{On}^2 x).$$

The primary eigenfunction Y_{E1} is taken as

$$Y_{E1} = \sin \pi z \sin \pi y/\eta$$

and now β_{E1}^2 takes the form

$$\beta_{E1}^2 = \frac{\pi^2\eta(1/4 + 1/4\eta^2)}{48/\pi^3 \times Pe \times I_2 \times 4/3\pi}$$

$$I_2 = \int_0^\eta \left[1 - \frac{\cosh(\pi y - \pi\eta/2)}{\cosh(\pi\eta/2)}\right] \sin^2(\pi y/\eta)\, dy \Bigg/ \left[1 - \frac{192}{\eta\pi^5}\tanh(\pi\eta/2)\right]. \quad (13)$$

The influence of the lateral walls is much more pronounced, and for smaller aspect ratios ($\eta \leqslant 1$) the thermal entry length will be considerably shorter. To illustrate this point, consider x_D (99% decay) for $\eta = 1/2$, 1 and ∞

$$x_D(1/2) = 0.14827Pe;$$

$$x_D(1) = 0.38496Pe;$$

$$x_D(\infty) = 0.612Pe.$$

In the limit $\eta \to \infty$, the effects of the lateral walls disappear, and the same result is recovered as for the adiabatic case. The thermal length is more than 2.5 times shorter for an aspect ratio of $\eta = 1/2$ than for $\eta = 1$. For $\eta = 1$ the entry length for adiabatic walls is twice as long as for conducting walls.

4. CONCLUSION

The thermal entry length problem was treated for the case where axial conduction is negligible. Explicit expressions for the thermal entry length x_D were derived, and it was shown that x_D is not very sensitive towards aspect ratio when the lateral walls are adiabatic. In the case of conducting walls however, the dependence becomes much stronger. If the lateral walls are weakly conducting, a value can be expected that lies between the two limits discussed here.

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